Maximum Power Transfer theorem
E & M BSC (H) PHYSICS SEM II

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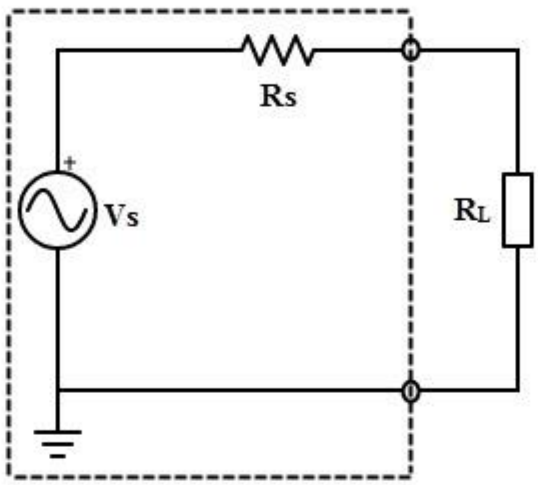
Why?

In any electric circuit, the electrical energy from the supply is supplied to the load where it is transformed into work. Practically, the complete power supplied will not present at load due to the heating effect (resistance) and other constraints in the circuit. Consequently, there exists a certain difference amongst supplying and delivering powers.

The power transferred to source always depend upon the load resistance i.e. changes with value of load resistance. The objective of Maximum power theorem is to find the condition where maximum power is transferred to load from supply.

How?

Statement: It states that in a linear, bilateral DC network, maximum power is transferred to the load when the load resistance becomes equal to the internal resistance of a source.

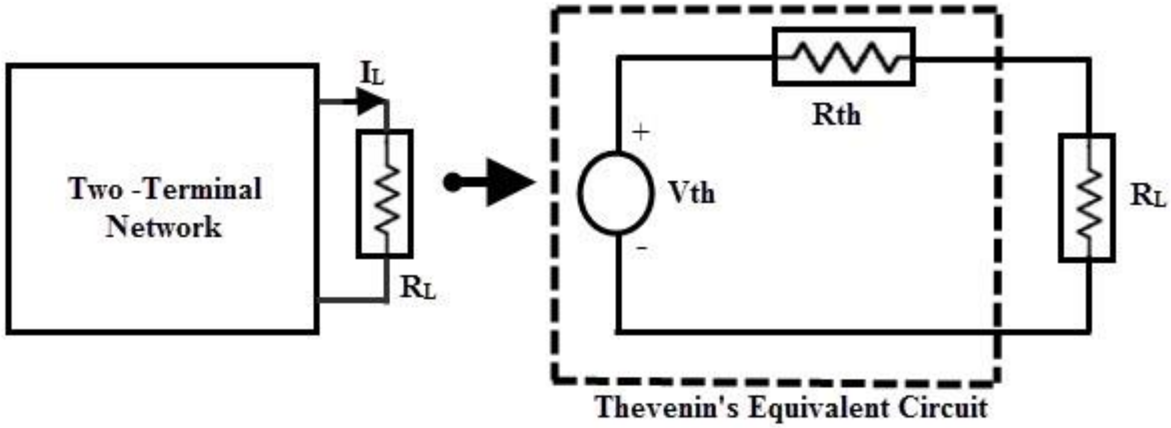


- If the power supply is voltage source then for maximum power transfer, the series resistance within source must be equal to the load resistance.
- However, in case of current power source, its parallel internal resistance must be equal to load resistance for maximum power transfer.

Let us prove the statement!

Proof:

If we have two terminal network, although we can use MPT directly, but in previous section we have studied Thevenin's Theorem, which converts any complex network to a simple network with Thevenin equivalents.



The original two terminal network is replaced with a Thevenin's equivalent circuit across the variable load resistance. The current through the load for any value of load resistance will be

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

The power absorbed by the load is

$$P_L = I_L{}^2 \times R_L$$

$$= \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L$$

Thus, the power transferred is dependent on R_L and R_{Th}. However, R_{Th} is constant, which means the power transferred is entirely dependent on load resistance, R_L.

To find the exact value of R_L, we apply differentiation to P_L with respect to R_L and equating it to zero as

$$\frac{dP(R_L)}{dR_L} = V_{Th}{}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L \times (R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

$$\Rightarrow (R_{Th} + R_L) - 2R_L = 0$$

$$\Rightarrow R_L = R_{Th}$$

Thus, maximum power transfers when load resistance is equal to Thevenin's equivalent resistance.

By substituting the R_{th} = R_L, we get

The maximum power delivered to the load is,

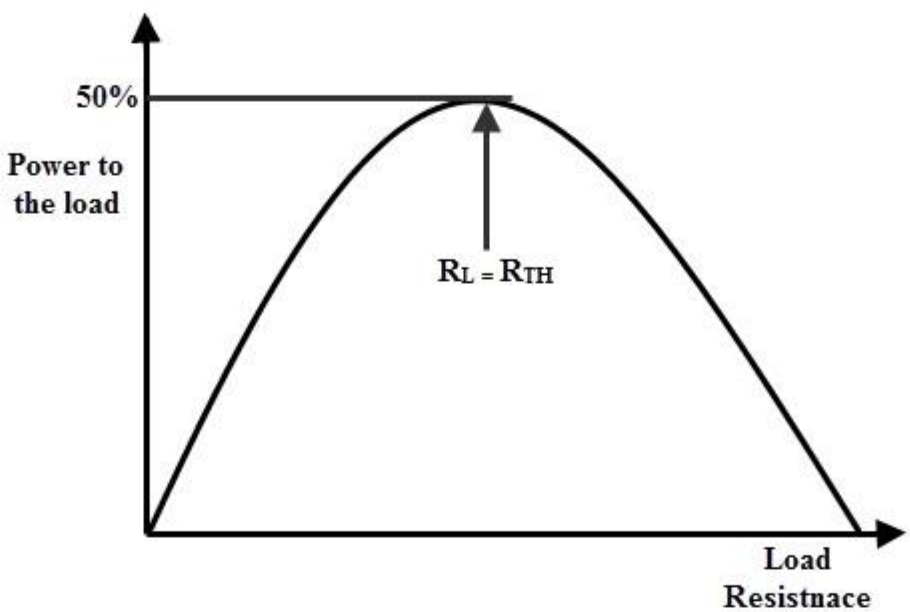
$$P_{max} = \left[\frac{V_{Th}}{R_{Th}+R_L}\right]^2 \times R_L\Bigg|_{R_L=R_{Th}}$$

$$= \frac{V_{Th}^{\ 2}}{4R_{Th}}$$

Total power transferred from source is

$$P_T = I_L^{\ 2}(R_{TH} + R_L)$$
$$= 2\, I_L^{\ 2} R_L$$

Curve of power delivered to the load with respect to the load resistance:



- When, load is zero, power transferred is equal to zero, as there is no voltage drop across load at that point.
- When R_{th} = R_L, maximum power is transfereed.
- The power is zero as the load resistance reaches to infinity as there is no current flow through the load.
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Is Efficiency is same as power transferred?
We must remember that this theorem results maximum power transfer but not a maximum efficiency.
Efficiency under the condition of maximum power transfer is

$$\textbf{Efficiency} = \text{Output} / \text{Input} \times 100$$
$$= I_L^2 R_L / 2 I_L^2 R_L \times 100$$
$$= 50\ \%$$

Hence, at the condition of maximum power transfer, the efficiency is 50%, that means a half percentage of generated power is delivered to the load and at other conditions small percentage of power is delivered to the load

Applications:
For some applications, it is desirable to transfer maximum power to the load than achieving high efficiency such as in amplifiers and communication circuits.

Application of MPT to DC Network:

Steps for Solving Network Using Maximum Power Transfer Theorem

- **Step 1 –** Remove the load resistance of the circuit.

- **Step 2 –** Find the Thevenin's resistance (R_{TH}) of the source network looking through the open circuited load terminals.

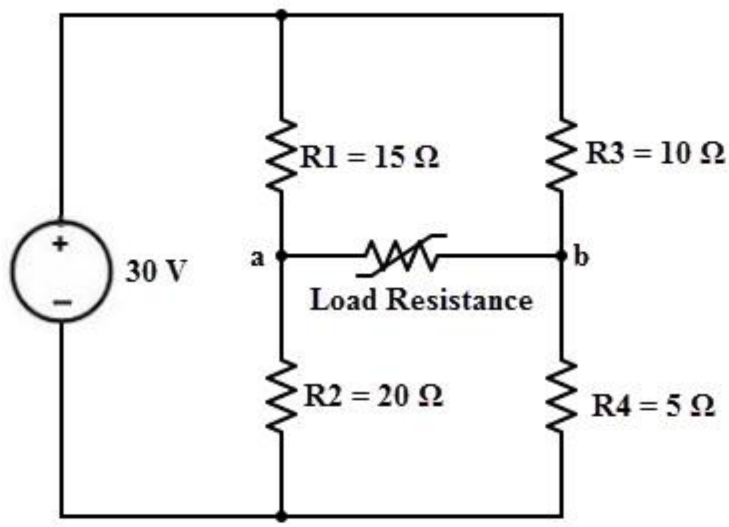
- **Step 3 –** As per the maximum power transfer theorem, this R_{TH} is the load resistance of the network, i.e., $R_L = R_{TH}$ that allows maximum power transfer.

- **Step 4 –** Maximum Power Transfer is calculated by the equation shown below

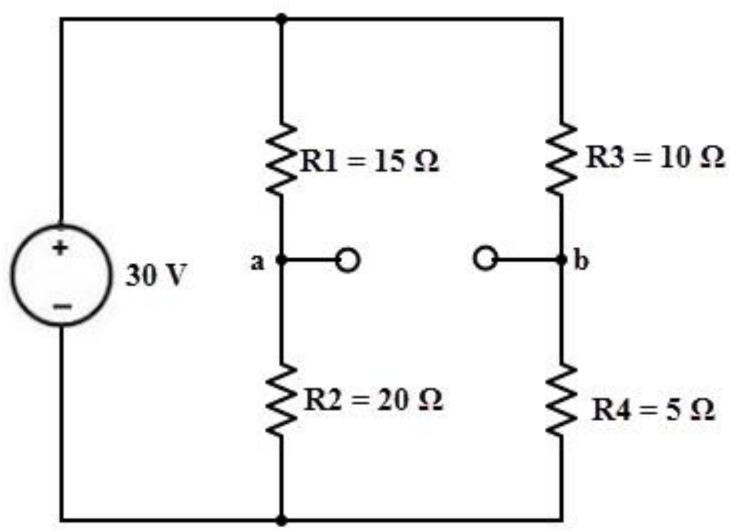
$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

Example:

Consider the below circuit to which we determine the value of the load resistance that receives the maximum power from the supply source and the maximum power under the maximum power transfer condition



- Disconnect the load resistance from the load terminals a and b. To represent the given circuit as Thevenin's equivalent, we are to determine the Thevenin's voltage V_{TH} and Thevenin's equivalent resistance R_{TH}.



The Thevenin's voltage or voltage across the terminals ab is $V_{ab} = V_a – V_b$

Va = V × R2 / (R1 + R2)

= 30 × 20 /×(20 + 15)

= 17.14 V

$$Vb = V \times R4 / (R3 + R4)$$

$$= 30 \times 5 / (10 + 5)$$

$$= 10 \text{ V}$$

$$Vab = 17.14 – 10$$

$$= 7.14 \text{ V}$$

$$V_{TH} = Vab = 7.14 \text{ Volts}$$

- Calculate the Thevenin's equivalent resistance R_{TH} by replacing sources with their internal resistances (here assume that voltage source has zero internal resistance so it becomes a short circuited).

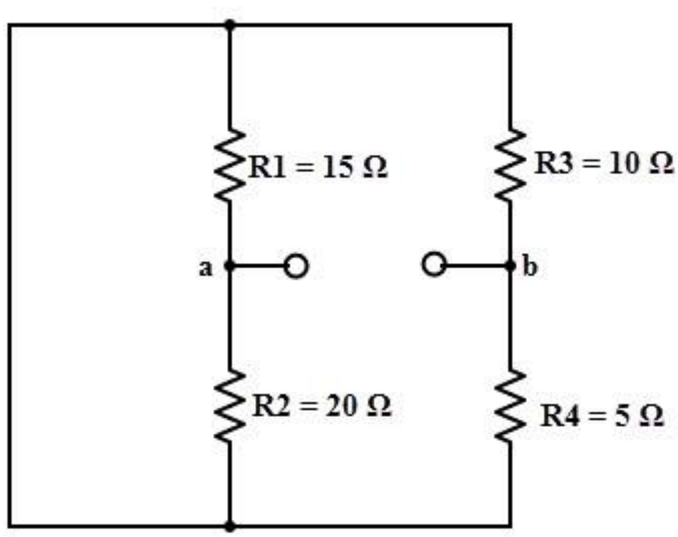
  Thevenin's equivalent resistance or resistance across the terminals ab is

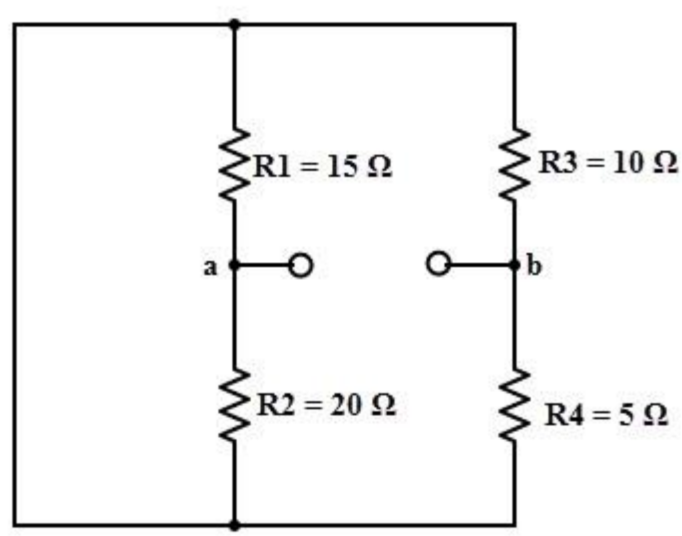
$$R_{TH} = Rab = [R1R2 / (R1 + R2)] + [R3R4 / (R3 + R4)]$$

$$= [(15 \times 20) / (15 + 20)] + [(10 \times 5) / (10 + 5)]$$

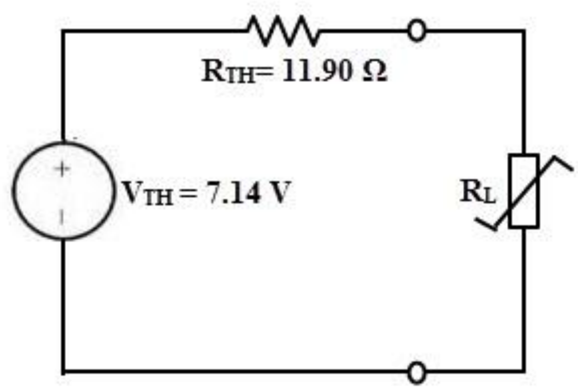
$$= 8.57 + 3.33$$

$$R_{TH} = 11.90 \text{ Ohms}$$





- Draw the Thevenin's equivalent circuit with above calculated values by reconnecting the load resistance:



- Use maximum power transfer theorem, $R_L = R_{TH} = 11.90$ Ohms
  And the maximum power transferred under this condition is,

$$P_{max} = V^2_{TH} / 4\,R_{TH}$$
$$= (7.14)^2 / (4 \times 11.90)$$
$$= 1.07 \text{ Watts}$$

Source:
www.electrical4u.com
www.electronicshub.org
Electrical Circuits by Ghosh